Seismic Analysis of Multistorey Building with Floating Column

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Abstract: In present scenario buildings with Floating Column is a typical feature in the modern multi-storey construction in urban India. Such features are highly undesirable in building built in seismically active areas. This study highlights the importance of explicitly recognizing the presence of the Floating Column in the analysis of building. Alternate measures, involving stiffness balance of the first storey and the storey above, are proposed to reduce the irregularity introduced by the Floating Columns. FEM analysis carried for 2D multi storey frames with and without floating column to study the responses of the structure under different earthquake excitation having different frequency content keeping the PGA and time duration factor constant. The time history of roof displacement, inter storey drift, base shear, column axial force are computed for both the frames with and without Floating Column.

Keywords: Floating column, stiffness balance, FEM codes, earthquake excitation, time history, roof displacement, Inter storey drift, base shear, column axial force.

1. INTRODUCTION

Many urban multistory buildings in India today have open first storey as an unavoidable feature. This is primarily being used to accommodate parking or reception lobbies in the first storey. Whereas the total seismic base shear as experienced by a building during an earthquake is dependent on its natural period, the seismic force distribution is dependent on the distribution of stiffness and mass along the height. The behavior of a building during earthquakes depends mainly on its overall shape, size and geometry, in addition to how the earthquake forces are carried to the ground. The earthquake forces developed at different floor levels in a building need to be brought down along the height to the ground by the shortest path; any deviation or discontinuity in this load transfer path results in poor performance of the building. Buildings with vertical setbacks (like the hotel buildings with a few storey wider than the rest) cause a sudden jump in earthquake forces at the level of discontinuity. Buildings that have fewer columns or walls in a particular storey or with unusually tall storey tend to damage or collapse which is initiated in that storey. Many buildings with an open ground storey intended for parking collapsed or were severely damaged in Gujarat during the 2001 Bhuj earthquake. Buildings with columns that hang or float on beams at an intermediate storey and do not go all the way to the foundation, have discontinuities in the load transfer path.

Conventional Civil Engineering structures are designed on the basis of strength and stiffness criteria. The strength is related to ultimate limit state, which assures that the forces developed in the structure remain in elastic range. The stiffness is related to serviceability limit states which assures that the structural displacement remains with the permissible limits. In case of earthquake forces the demand is for ductility. Ductility is an essential attribute of a structure that must respond to strong ground motions. Ductility is the ability of the structure to undergo distortion or deformation without damage or failure which results in dissipation of energy. Larger is the capacity of the structure to deform plastically without collapse, more is the resulting ductility and the energy dissipation. This causes reduction in effective earthquake forces. Most of the energy developed during earthquake is dissipated by columns of the soft stories. In this process the plastic hinges are formed at the ends of columns, which transform the soft storey into a mechanism. In such case the collapse is unavoidable. Therefore, the soft stories deserve a special consideration in analysis and design.

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2. PRESENT STUDY

The objective of the present work is to study the behavior of multistory buildings with and without floating columns under earthquake excitations. RC Frames of different stiffness on floor wise and height of building are considered. The base of the building frame is assumed to be fixed. The time history analysis of these RC Frames has been done by subjecting the whole system to BHUJ earthquake ground motion, using FEM Package SAP2000.

2.1 Scope of Present Work

In present study, an attempt has been made to study following aspects

1) Modelling of multistorey building frames with and without floating column using finite element software, SAP2000.

2) The column sizes having different dimensions are modelled from ground level to the top storey level.

3) Dynamic analysis is done by Time History method is carried out for all the models.

4) Comparative study is made for all frames with and without floating column.

5) Study on the variations in the structural response due to the earthquake motions are tabulated

2.2 Analysis

Steps to be Followed for the Analysis of RC Frame:

Step 1:- Discretising the domain: Dividing the element into number of nodes and numbering them globally i.e. breaking down the domain into smaller parts.

Step 2:- Writing of the Element stiffness matrices: The element stiffness matrix or the local stiffness matrix is found for all elements and the global stiffness matrix of size 3n x 3n is assembled using these local stiffness matrices.

Step 3:- Assembling the global stiffness matrices: The element stiffness matrices are combined globally based on their degrees of freedom values.

Step 4:- Applying the boundary condition: The boundary element condition is applied by suitably deleting the rows and columns which are not of our interest.

Step 5:- Solving the equation: The equation is solved in SAP2000 to give the value of U.

Post- processing: The reaction at the support and internal forces are calculated.

There are three types of analysis performed

1) Static Analysis:

- Structural analysis is the process to analyze a structural system to predict its responses and behaviors by using physical laws and mathematical equation.
- The main objective of structural analysis is to determine internal forces, stresses and deformation of structures under various load effect.

2) Dynamic Analysis:

- Dynamic analysis of structure is a part of structural analysis in which behaviour of flexible structure subjected to dynamic loading is studied.
- Dynamic load always changes with time.
- Dynamic load comprises of wind, live load, earthquake load etc. Thus in general we can say almost all the real life problems can be studied dynamically.

If dynamic loads changes gradually the structure's response may be approximately by a static analysis in which inertia forces can be neglected. But if the dynamic load changes quickly, the response must be determined with the help of dynamic analysis in which we cannot neglect inertial force which is equal to mass time of acceleration (Newton's 2nd law).

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Mathematically F = M x a

Where F is inertial force, M is inertial mass and 'a' is acceleration.

3) Time History Analysis:

A linear time history analysis overcomes all the disadvantages of modal response spectrum analysis, provided non-linear behaviour is not involved. This method requires greater computational efforts for calculating the response at discrete time. One interesting advantage of such procedure is that the relative signs of response qualities are preserved in the response histories. This is important when interaction effects are considered in design among stress resultants.

Here dynamic response of the plane frame model to specified time history compatible to IS code spectrum and Bhuj earthquake has been evaluated.

The equation of motion for a multi degree of freedom system in matrix form can be expressed as:

$$[m]\{\vec{x}\} + [c]\{\vec{x}\} + [k]\{x\} = -x\vec{g}(t)[m]\{I\}$$
(5)

Where,

[*m*]= mass matrix

[k] = stiffness matrix

[*c*]= damping matrix

 $\{I\}$ = unit vector

 $x\dot{g}(t)$ = ground acceleration

The mass matrix of each element in global direction can be found out using following expression:

$$m = [T^{T}][m_{e}][T]$$

$$[m_{e}] = \frac{\rho A L}{420} \begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ 0 & 156 & 22L & 0 & 54 & -13L \\ 0 & 22L & 4L^{2} & 0 & 13L & -3L^{2} \\ 70 & 0 & 0 & 140 & 0 & 0 \\ 0 & 54 & 13L & 0 & 156 & -22L \\ 0 & -13L & -3L^{2} & 0 & -22L & 4L^{2} \end{bmatrix}$$

$$[T] = \begin{bmatrix} C & S & 0 & 0 & 0 & 0 \\ -S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C & S & 0 \\ 0 & 0 & 0 & 0 & -S & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(8)$$

 $(\cap$

The solution of equation of motion for any specified forces is difficult to obtain, mainly due to coupling variables $\{x\}$ in the physical coordinate. In mode superposition analysis or a modal analysis a set of normal coordinates i,e principal coordinate is defined, such that, when expressed in those coordinates, the equations of motion becomes uncoupled. The physical coordinate $\{x\}$ may be related with normal or principal coordinates $\{q\}$ from the transformation expression as,

 $\{x\} = [\Phi] \{q\}$

(9)

 $[\Phi]$ Is the modal matrix

Time derivative of {x} are

$$\{x'\} = [\Phi] \{q'\}$$

 $\{\vec{x}\} = [\Phi] \{\vec{q}\}$

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Substituting the time derivatives in the equation of motion, and pre-multiplying by $[\Phi]$ T results in,

$$[\Phi]^{T}[m][\Phi]{\dot{q}} + [\Phi]^{T}[c][\Phi]{\dot{q}} + [\Phi]^{T}[k][\Phi]{\dot{q}} = (-\dot{x_{g}}(t)[\Phi]^{T}[m]{I})$$

More clearly it can be represented as follows:

 $[M]{\ddot{q}}+[C]{\dot{q}}+[K]{q}={\text{Peff}(t)}$

Where,

 $[M] = [\Phi]^T[m][\Phi]$

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[C] = [\Phi]^T[c][\Phi] = 2 \zeta [M] [\omega]
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 $[K] = [\Phi]^T [k] [\Phi]$

 $\{P_{eff}(t)\} = (-\dot{x}_{g}(t)[\Phi]^{T}[m]\{l\})$

[M], [C] and [K] are the diagonalised modal mass matrix, modal damping matrix and modal stiffness matrix, respectively, and {Peff(t)} is the effective modal force vector.

3. RESULTS AND DISCUSSIONS

A comparative study is carried out both for 2D and 3D RC frame structures with and without floating columns. A study is carried out to find variation in time period and structural response for various parameters like floor displacement, base shear, shear force, bending moment and torsion for the beams and axial force for all the models. The time histories is presented and comparison are made with those obtained from the analysis all the type of frames.

3.1 2D TYPE-1

In this type the structure is modelled with 2D G+3 Frame without floating column and with floating column. Both column and beam sizes are 200X450. The comparison is carried out for different models, where the models are modelled by varying column sizes 200X450 to 200X600 from ground to top floor model to model.

The Figure no.6.1 & 6.2 shows the frame with and without floating columns. The load path in 2D Type-1 model is diverted by removing the central column of ground floor.





(10)

Fig-3.1 2D Type-1 Frame without Floating Column

Fig-3.2 2D Type-1 Frame with Floating Column

3.1.1 Variation in Natural Period

The variation in natural period due to the effect of floating column is studied on 2D Type-1 frame modeled with four storey assuming foundation is fixed. The frame is analyzed with 2D frame analysis by using SAP2000 and the results are tabulated in Table 3.1 and 3.2.

	2D TYPE-1 FR	2D TYPE-1 FRAME WITHOUT FLOATING COLUMN						
Mode No:	GF to TPF - 200X450	GF-200X600 FF to TF - 200X450	GF to FF- 200x600 SF to TPF- 200X450 TPF- TPF- TPF-	GF to SF- 200x600 TF to TPF- 200X450 TPF- TPF- TPF-	GF to TPF- 200X600			
Mode-1	0.349	0.322	0.308	0.305	0.303			
Mode-2	0.110	0.102	0.101	0.098	0.093			
Mode-3	0.062	0.058	0.056	0.053	0.049			

 Table-3.1: Fundamental Period for 2D Type-1 Frame without Floating Column

	2D TYPE-1 FRAME WITH FLOATING COLUMN						
Mode No:	GF to TPF - 200X450	GF-200X600 FF to TF - 200X450	GF to FF- 200x600 SF to TPF- 200X450	GF to SF- 200x600 TF to TPF- 200X450 TPF- TPF- TPF-	GF to TPF- 200X600		
Mode-1	0.379	0.341	0.325	0.323	0.320		
Mode-2	0.118	0.113	0.114	0.115	0.114		
Mode-3	0.114	0.109	0.107	0.103	0.099		

The results obtained for Modal time period are shown in Table 3.1 and 3.2. Introduction of floating column in the RC frame increases the time period of bare frames due to decrease in the stiffness of structure. It has been found that by incorporating floating column in Type 1 model there was 7.92% increase in time period compared to without floating column. And in all other for increase in column size (i.e. Ground Floor 200X600 and First Floor to Top Floor 200X450, Ground Floor to First Floor 200X600 and Second Floor to Top Floor 200X450, Ground Floor to Second Floor 200X600 and Top Floor 200X450 and Ground Floor to Top Floor 200X600) four models with decrease of 7.73%,11.74%, 12.60% and 13.18% in modal period was observed when compared to model Ground Floor to Top Floor 200X450 with floating column.

Similarly for all other increase in column size (i.e. Ground Floor 200X600 and First Floor to Top Floor 200X450, Ground Floor to First Floor 200X600 and Second Floor to Top Floor 200X450, Ground Floor to Second Floor 200X600 and Top Floor 200X450 and Ground Floor to Top Floor 200X600) four models with decrease of 10.02%,14.27%, 14.77% and 15.56% in modal period was observed when compared to model Ground Floor to Top Floor 200X450 with floating column.

3.1.2 Variation in Base Shear

The variation in Base Shear as per IS 1893-2002 due to the effect of floating column is studied on 2D Type-1 frame modelled with four storey assuming foundation is fixed. The frame is analyzed with 2D frame analysis by using SAP2000 and the results are tabulated in Table 3.3 and 3.4.

	2D TYPE-1 FRAME WITHOUT FLOATING COLUMN				
BASE SHEAR AS PER IS 1893-2002 IN (KN)	GF to TPF - 200X450	GF-200X600 FF to TF - 200X450	GF to FF- 200X600 SF to TPF- 200X450	GF to SF- 200X600 TF to 200X450	GF to TPF- 200X600
	19.373	18.564	19.392	20.721	21.824
Percentage Variation	-	-4.36	0.10	6.51	11.23

 Table-3.3: Base Shear for 2D Type -1 Frame without Floating Column

	2D TYPE-1 FRAME WITH FLOATING COLUMN				
BASE SHEAR AS PER IS 1893-2002 IN (KN)	GF to TPF - 200X450	GF-200X600 FF to TF - 200X450	GF to FF- 200X600	GF to SF- 200X600 TF to TPF- 200X450	GF to TPF- 200X600
	22.144	21.236	22.066	23.534	24.744
Percentage Variation	-	-4.28	-0.35	5.91	10.50





Fig: 3.3: Variation in Base Shear for 2D TYPE-1 model without Floating Column



Fig: 3.4: Variation in Base Shear for 2D TYPE-1 model with Floating Column.

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Fig: 3.5: Variation in Base Shear for 2D TYPE-1 model without Floating Column.



Fig: 3.6: Variation in Base Shear for 2D TYPE-1 model with Floating Column.

3.1.3 Variation in Roof Displacement

The variation in Roof Displacement due to the effect of floating column is studied on 2D Type-1 frame modeled with four storey assuming foundation is fixed. The frame is analyzed with 2D frame analysis by using SAP2000 and the results are tabulated in Table 3.5and 3.6.

	2D TYPE-1 FRAME WITHOUT FLOATING COLUMN					
ROOF DISPLA- CEMENT IN (mm)	GF to TPF - 200X450	GF-200X600 FF to TF - 200X450	GF to FF- 200X600 SF to TPF- 200X450	GF to SF- 200X600 TF to TPF- 200X450 TPF- TPF- TPF-	GF to TPF- 200X600	
	5.4	4.7	4.4	4.3	4.2	
Percentage Variation	-	12.96	18.52	20.37	22.22	

 Table-3.5: Roof Displacement for 2D Type -1 Frame without Floating Column.

	2D TYPE-1 FRAME WITHOUT FLOATING COLUMN				
ROOF DISPLA- CEMENT IN (mm)	GF to TPF - 200X450	GF-200X600 FF to TF - 200X450	GF to FF- 200X600 SF to TPF- 200X450	GF to SF- 200X600 TF to TPF- 200X450	GF to TPF- 200X600
	6.9	5.8	5.4	5.3	5.2
Percentage Variation	-	15.94	21.73	23.18	24.63

Table-3.6: Roof Displacement for 2D Type -1 Frame with Floating Column.



Fig: 3.7: Variation in Roof Displacement for 2D TYPE-1 model without Floating Column.





3.1.4 Variation in Column Axial Force

The variation in Column Axial Force due to the effect of floating column is studied on 2D Type-1 frame modeled with four storey assuming foundation is fixed. The frame is analyzed with 2D frame analysis by using SAP2000 and the results are tabulated in Table 3.7and 3.8.

Floors	GF-TPF 200X450	GF 200X600	GF-FF 200X600	GF-SF 200X600	GF-TPF 200X600
Base	61.097	62.141	64.858	67.991	71.534
Percentage Variation	-	1.68	5.79	10.13	14.59
1^{st}	54.93	53.069	55.786	58.919	62.462
Percentage Variation	-	-3.50	1.53	6.77	12.05
2^{nd}	37.571	37.253	37.642	40.323	43.600
Percentage Variation	-	-0.85	0.18	6.82	13.82
3 rd	21.034	20.993	21.038	21.448	24.471
Percentage Variation	-	-0.19	0.01	1.93	14.04
4 th	6.068	6.052	6.083	6.091	6.750
Percentage variation	-	-0.26	0.24	0.37	10.10

Table-3.7: Column Axial Force in (KN) for 2D Type -1 Frame without Floating Column

Table-3.8: Column Axial Force in (KN) for 2D Type -1 Frame with Floating Column

	GE_TPE				GE-TPE
Floors		GF 200X600	GF-FF 200X600	GF-SF 200X600	
	200X450				200X600
Base	92.963	94.007	98.032	98.039	106.835
Percentage		1.11	C 17	5.17	12.00
Variation	-	1.11	5.17	5.17	12.98
1^{st}	86.159	84.935	88.960	88.970	97.763
Percentage		1.42	2.14	2.16	11.07
Variation	-	-1.43	3.14	5.10	11.87
2^{nd}	60.922	60.214	61.216	61.226	69.468
Percentage		1 17	0.49	0.40	12.20
Variation	-	-1.17	0.48	0.49	12.50
3 rd	35.748	35.560	35.714	35.726	41.014
Percentage		0.52	0.00	0.06	12.92
Variation	-	-0.32	-0.09	-0.06	12.85
4 th	12.573	12.490	12.618	12.635	14.393
Percentage		0.66	0.25	0.40	12 64
Variation	-	-0.00	0.55	0.49	12.04



Fig: 3.9: Variation in Column Axial Force for 2D TYPE-1 model without Floating Column.

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Fig: 3.10: Variation in Column Axial Force for 2D TYPE-1 model with Floating Column.

3.1.5 Variation in Storey Drift

The Storey Drift for Type-1, 2D Frames by increasing the column size from ground floor to top floor is compared. The maximum Storey Drift is obtained from SAP2000 software as follows. It is observed that the maximum inter storey drift decreases with increasing the column size.



Fig: 3.11: Comparison of predicted Storey Drift (mm) of Type-1, 2D Frame without Floating Column obtained from SAP2000 software by increasing the column size from Ground floor to Top floor.

 Table 3.9: Comparison of predicted Storey Drift (mm) of Type-1 , 2D frame without Floating Column obtained by increasing the column size from ground floor to top floor.

Size of the column (mm)	Storey Drift Ratio (mm)	% Decrease
GF-TPF (200X450)	4.31	
GF(200X600)	3.92	9.04
GF-FF (200X600)	3.30	23.43
GF-SF (200X600)	3.28	23.90
GF-TPF (200X600)	3.47	19.50

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Fig: 3.12: Comparison of predicted Storey Drift (mm) of Type-1, 2D frames with Floating Column obtained from SAP2000 software by increasing the column size from Ground floor to Top floor.

 Table 3.10: Comparison of predicted Storey Drift (mm) of Type-1, 2D frames with Floating Column obtained by increasing the column size from ground floor to top floor.

Size of the column (mm)	Storey Drift Ratio (mm)	% Decrease
GF-TPF (200X450)	5.35	
GF (200X600)	4.60	14.01
GF-FF (200X600)	3.72	30.46
GF-SF (200X600)	3.88	27.47
GF-TPF (200X600)	4.09	23.55

4. CONCLUSIONS

The behaviour of multistory building with and without floating column is studied under different earthquake excitation. The compatible time history and Bhuj earthquake data has been considered. The static and free vibration results obtained using present finite element code are validated. The dynamic analysis of frame is studied by varying column size dimension. It is concluded that by increasing the column size the maximum displacement and inter storey drift values are reducing.

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